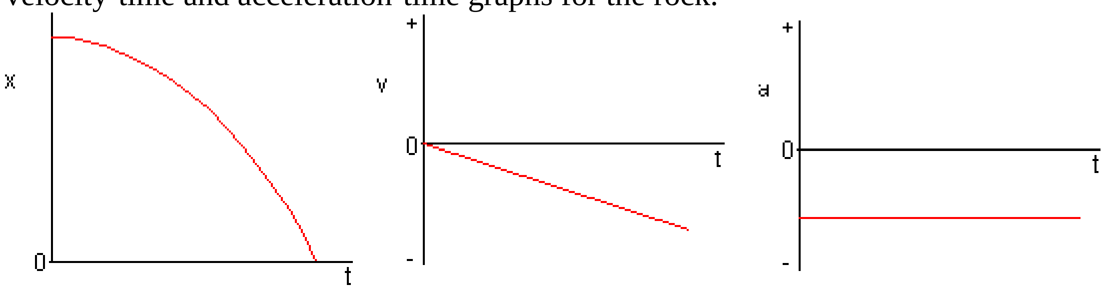
Try-Math-A-Lon

Fall Regional Competition

Engineering Contest 2008 Solutions

Problem 1 Solution. A rock is dropped off a bridge and hits the water 2.3 seconds later. The rock accelerates due to gravity at a rate of 10 m/s^2.
a. Using the water as the zero position and calling the upward direction positive, draw position-time, velocity-time and acceleration-time graphs for the rock.



b. What is the rock's speed just before hitting the water?
Knowns: acceleration, initial velocity, time
Unknown: final velocity
Appropriate mathematical model: v_f = acc.(change in time) + v_i
v_f = acc.(change in time) + v_i = (-10 m/s^2)(2.3 s) + 0 m/s = -23 m/s
The negative signs indicate that the acceleration and the final velocity are directed toward the origin.

Problem2:

v=fλ

λ= v/f

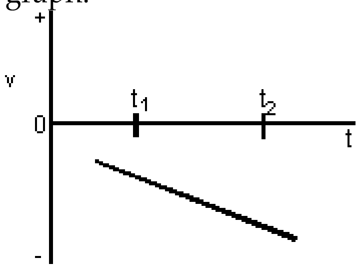
λ= <u>3 x 10^8 m/s</u>

2.2 x 10^6

λ= 1.4 x 10^2

Acceptable answers also : 140 m or 136m

Problem 3 Solution. Answer the following questions for the interval t_1 to t_2 based on the velocity-time graph.



a. Is the velocity positive, negative or zero? Explain.

The velocity is negative since it is in the negative region of the graph.

b. Is the velocity increasing, decreasing or not changing? Explain.

The *magnitude* of the velocity (speed) is increasing since the velocity is getting farther from zero.

c. Is the acceleration positive, negative or zero? Explain.

The acceleration is negative since the slope of the graph is negative. Also, since the object is speeding up (+) and traveling toward the origin (-) we know the acceleration is negative.

d. Is the acceleration increasing, decreasing or constant? Explain.

The acceleration is constant since the slope of the v-t graph remains constant.

e. Is the displacement for the time interval positive, negative or zero? Explain.

The displacement is negative (in this case the area "beneath" the curve is the region between the graphed line and the time axis.) A negative displacement means motion toward the negative end of the coordinate axis.

f. Is the position increasing, decreasing or zero? Explain.

The position is decreasing, which means that motion is toward the negative end of the numberline.

g. Is the position positive, negative or zero? Explain.

It is not possible to tell the starting point from the velocity-time graph. Different starting points could result in positive, negative or zero positions at, say, t_2.

h. Describe a motion consistent with this graph.

Your hot honey is standing near the motion detector. Upon seeing your lover, you run with a constant acceleration in his/her direction. Fortunately, the graph doesn't tell us what happens after that.

Problem 4:

Note: To simplify calculations, an approximated value of g is often used - 10 m/s/s. Answers obtained using this approximation are shown in parenthesis.

F_{norm} = 80 N; m = 8.16 kg; F_{net} = 40 N, right; a = 4.9 m/s/s, right

(F_{norm} = 80 N; m = 8 kg; F_{net} = 40 N, right; a = 5 m/s/s, right)

Since there is no vertical acceleration, normal force = gravity force.

The mass can be found using the equation $F_{grav} = m \cdot g$.

The F_{net} is the vector sum of all the forces: 80 N, up plus 80 N, down equals 0 N. And 50 N, right plus 10 N, left = 40 N, right.

Finally, a = F_{net} / m = (40 N) / (8.16 kg) = 4.9 m/s/s.

Problem 5:

<u>Problem A</u>: Computing the Volume of the bucket

The excavator used in this study has a maximum capacity of 3 metric tones that includes the weight of the empty bucket plus the weight of material. Assuming that the empty bucket weights 25% of its total maximum load, compute in m³ the volume of the bucket based on a full load of wet earth with a density of 2,698 lb/yd³.

Answer space:

*Let call **W** the total mass of the bucket. The mass of the content of the bucket when filled is*

$$M = (1 - 0.25)W \qquad \text{(eq.1)}$$

Let call ρ the density of the earth and V the volume of the bucket. The mass M is expressed as:

$$M = \rho V \text{ there for } V = M/\rho \qquad \text{(eq.2)}$$

Conversion of density: $$\rho = \frac{2{,}698 lb}{yd^3} \times \frac{0.452 kg}{lb} \times \frac{1 yd^3}{\left(36 in/yd \times 0.0254 m/in\right)^3} = 1595\, kg/m^3$$

$$V = \frac{0.75W}{\rho} = \frac{0.75 * 3000\, kg}{1595\, kg/m^3} = 1.410\, m^3 \qquad \text{(eq.3)}$$

As shown on figure 1, several forces act on the bucket during the scooping process. The main once represented here are:

- $\vec{P}$ The weight of the bucket and rock in it;
- $\vec{F}_1$ The pressure exerted by the pile of material inside the bucket;
- $\vec{F}_2$ The cutting and the friction resulting force on the edge of the bucket.

To simplify the study of these forces, each of them is decomposed into two main components: one horizontal on the ***X***-axis and a vertical one on the ***Y***-axis. The resulting force on the X-axis is named $\boldsymbol{F}_x$ and on the ***Y***-axis $\boldsymbol{F}_y$. A model of these two forces is given as follows:

$$\begin{cases} F_x(x) = \dfrac{k_x}{1+(x^2-0.25)^4} \\ F_y(y) = k_y\left(b_3\sigma^3 + b_2\sigma^2 + b_1\sigma + b_0\right) \end{cases} \qquad \text{(eq. II.1)}$$

Where σ is a variable and $b_{0...3}$, k_x and k_y are constants to be determined later.

<u>Problem B</u>: Computing the maximum forces on ***X***-axis

The maximum value of $F_x(x)$ is found by solving the following equation:

$$8x(x^2-0.25)^3 = 0 \qquad \text{(eq.II.2)}$$

Considering that the pile of material starts a $x=0$ find the value of x that leads to the maximum value of F_x for $k_x = 10000\ N$. Justify your selection and compute the resulting value of F_x.

Answer space:

Resolving the equation: $8x(x^2-0.25)^3 = 0$

(eq.4)

$$8x(x^2-0.25)^3 = 0 \Rightarrow \begin{cases} x = 0 \\ or \\ (x^2-0.25)^3 = 0 \Rightarrow (x^2-0.25) = 0 \Rightarrow x^2 = 0.25 \Leftrightarrow x = \pm\sqrt{0.25} \end{cases}$$

$$\begin{cases} x = 0 \\ x = 0.5 \\ x = -0.5 \end{cases} \qquad \textit{(eq.5)}$$

Since the pile of material starts at $x=0$ *there is no material before that. Therefore a negative value of* x *is not admissible. The maximum value of* $\boldsymbol{F_x}$ *is given by either x=0 or x=0.5.*
Let compute $\boldsymbol{F}_x$ *for both values.*

$$F_x(x=0) = k\left(\frac{1}{1+(0-0.25)^4}\right) = k\left(\frac{1}{1+0.00390625}\right) = \frac{10000}{1.00390625} = 9961.089N$$

$$F_x(x=0.5) = k\left(\frac{1}{1+((0.5)^2-0.25)^4}\right) = k\left(\frac{1}{1+(0.25-0.25)^2}\right) = \frac{k}{1} = 10000N$$

$$\begin{cases} x = 0.5 \\ and \\ F_x = 10000N \end{cases} \qquad \textit{(eq.6)}$$